VIDYAPEETH ACADEMY
SINCE 2013
IIT JEE | NEET | FOUNDATION

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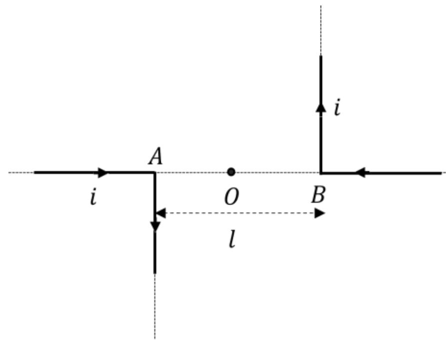
JEE Main 2023 (Memory based)

29 January 2023 - Shift 1

Answer & Solutions

PHYSICS

1. Point O and two long wires are kept in same plane such that point O lies at middle of the line. Then magnetic field at point O due to the current i flowing in both the wires is equal to



A. $\frac{\mu_o i}{2\pi l}$

B. $\frac{\mu_o i}{\pi l}$

C. $\frac{2\pi\mu_o i}{l}$

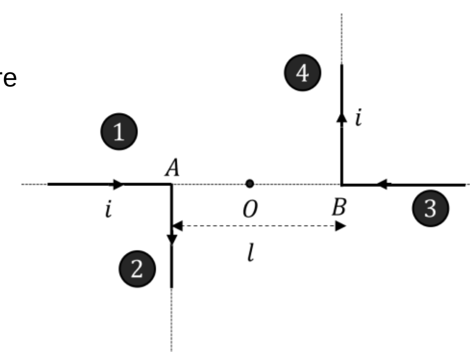
D. $\frac{\mu_o i}{2l}$

Answer (B)

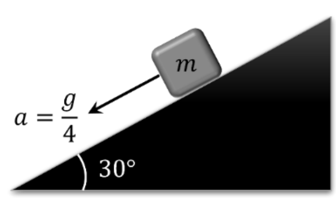
Solution:

Magnetic field due to Wire section (1) and (3) shown in figure will not generate a field at O.
Magnetic field due to Wire section (2) and (4) will be equal to;

$$B = 2 \times \frac{\mu_0 i}{4\pi\left(\frac{l}{2}\right)}(\hat{k}) = \frac{\mu_0 i}{\pi l}(\hat{k})$$



2. A block is sliding down an inclined plane of inclination $30°$, with an acceleration of $g/4$. Find the co-efficient of friction between the block and incline.



A. $\frac{1}{\sqrt{3}}$

B. $\frac{1}{2\sqrt{3}}$

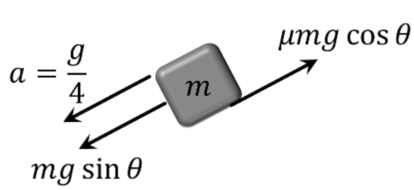
C. $\frac{1}{3}$

D. $\frac{1}{2}$

Answer (B)

Solution:

FBD for the given situation will be:



Balancing force gives:

$$mg \sin\theta - \mu mg \sin\theta = ma$$
$$\frac{g}{4} = g \sin\theta - \mu g \cos\theta$$
$$\mu = \frac{1}{2\sqrt{3}}$$

3. A car is moving on a circular track of radius $50\ cm$ with coefficient of friction being 0.34. On this horizontal track the maximum safe speed for turning is equal to $(g = 10\ \frac{m}{s^2})$

A. 1.03
B. 1.7
C. 1.3
D. 1.8

Answer (C)

Solution:

Friction will provide required centripetal acceleration to move in the circle:
So,
$$\frac{mv^2}{r} = \mu mg$$
$$v = \sqrt{\mu g R}$$
$$v = \sqrt{0.34 \times 10 \times \frac{1}{2}}$$
$$v = 1.3\ m/s$$

4. Find the ratio of maximum wavelength of $Lyman$ series of $Hydrogen$ atom to minimum wavelength of $Balmer$ series of $Helium$ ion.

A. $4/3$
B. 1
C. $3/2$
D. $3/4$

Answer (A)

Solution:

λ_{max} for Lyman series (for hydrogen atom)
ΔE will be for $n = 2$ to $n = 1$
$$\Delta E = 13.6 \times \left(1 - \frac{1}{4}\right)\ eV = \frac{3}{4} \times 13.6\ eV$$
$$\lambda_{max} = \frac{12400}{\frac{3}{4} \times 13.6}\ \text{Å}$$
λ_{min} for Balmer series (for He ion)
ΔE will be for $n = \infty$ to $n = 2$
$$\Delta E = 13.6 \times 4 \times \left(\frac{1}{4}\right)\ eV = 13.6\ eV$$
$$\lambda_{min} = \frac{12400}{13.6}\ \text{Å}$$
$$\frac{\lambda_{max}}{\lambda_{min}} = \frac{13.6}{\frac{3}{4} \times 13.6} = \frac{4}{3}$$

5. Find the excess pressure inside a soap bubble of radius 'R' and surface tension 'T'.

A. T/R
B. $2T/R$
C. $3T/R$
D. $4T/R$

Answer (D)

Solution:

We know that excess pressure inside soap bubble is:

$$(\Delta P)_{soap\ bubble} = \left(\frac{4T}{R}\right)$$

6. Two point masses (mass m each) are moving in a circle of radius R under mutual gravitational attraction. Find the speed of each mass.

A. $\sqrt{\frac{GM}{4R}}$
B. $\sqrt{\frac{GM}{2R}}$
C. $\sqrt{\frac{GM}{8R}}$
D. $\sqrt{\frac{GM}{R}}$

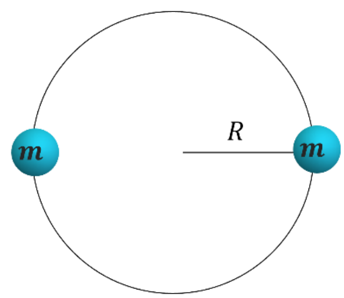
Answer (A)

Solution:

Masses will revolve around their center of mass,
Mutual gravitational force provides the centripetal force:

$$\frac{G \times m \times m}{(2R)^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{Gm}{4R}}$$

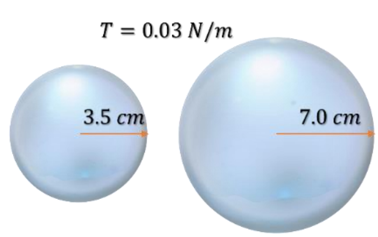


7. Find the work done in expanding the soap bubble from radius $r_1 = 3.5\ cm$ to $r_2 = 7.0\ cm$. (Given surface tension of soap solution, $T = 0.03\ N/m$)

A. $0.14\ mJ$
B. $1.4\ mJ$
C. $0.7\ mJ$
D. $2.8\ mJ$

Answer (D)

Solution:



Work done = Change in surface energy of soap bubble
Work done = $2T(4\pi r_2^2 - 4\pi r_1^2)$
$W = 2 \times 4\pi T(r_2^2 - r_1^2)$
$W = 2 \times 4\pi \times 0.03(7^2 - 3.5^2) \times 10^{-4}\,J$
$W = 2.8\,mJ$

8. In an $isochoric$ process on an ideal gas initial temperature is equal to $27°\,C$ with an initial pressure being equal to $270\,kPa$. Now if final temperature is made equal to $36°\,C$ then final pressure is equal to approximately.

A. $298\,kPa$
B. $270\,kPa$
C. $360\,kPa$
D. $278\,kPa$

Answer (D)

Solution:

For an iso-choric process (Volume is constant):

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$
$$\frac{270}{300} = \frac{P_f}{309}$$
$$P_f = 278.1\,kPa$$

9. If half-life of a sample is $30\,minutes$. Find the fraction of undecayed sample after $90\,minutes$.

A. $1/4$
B. $3/4$
C. $1/8$
D. $7/8$

Answer (C)

Solution:

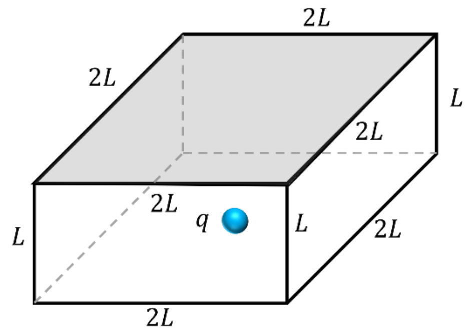
For first order decay:
$$N = N_0 e^{-\lambda t}$$

Half-life of a sample can be given as:
$$\lambda = \frac{ln2}{t_{\frac{1}{2}}}$$
$N =$ number of undecayed nuclei

$$N = N_0 e^{-\lambda \times 90}$$
$$N = N_0 e^{-\left(\frac{ln2}{30} \times 90\right)}$$
$$N = N_0 e^{-(ln8)}$$
$$N = \frac{N_0}{8}$$
$$\text{Fraction undecayed} = \frac{N}{N_0} = \left(\frac{1}{8}\right)$$

10. A charge q is placed at the centre of bottom face as shown:
Find the flux through the shaded surface.

A. $\frac{2q}{7\varepsilon_o}$
B. $\frac{q}{12\varepsilon_o}$
C. $\frac{q}{4\varepsilon_o}$
D. $\frac{q}{6\varepsilon_o}$



Answer (D)

Solution:

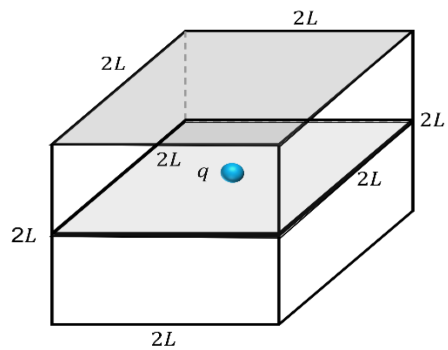
Place a similar box at the bottom as shown in figure. We get a cube of side $2L$. Charge q is then at the center of new cube.

From Gauss's Law:

$\phi = \left(\frac{q_{en}}{\epsilon_0}\right)$

From one surface of the cube, flux will be :

$\phi = \frac{1}{6}\left(\frac{q_{en}}{\epsilon_0}\right)$



11. Two coherent waves of amplitude $8\ cm$ each are superimposed on one another. If the amplitude of resultant wave is $8\ cm$ then the phase difference between two waves is equal to:

A. $2\pi/3$
B. $\pi/3$
C. $\pi/4$
D. $3\pi/4$

Answer (A)

Solution:

Given: $A_1 = A_2 = 8\ cm$
$A_R = 8\ cm$

Resultant Amplitude can be given as:

$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$

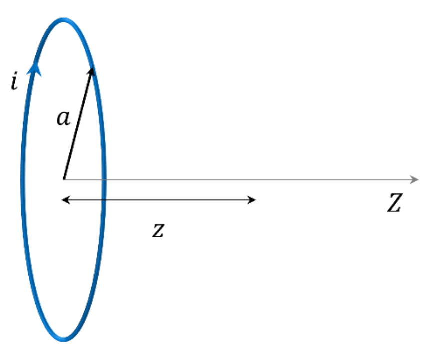
$8 = \sqrt{64 + 64 + 128 \cos\phi}$

$\cos\phi = -1/2$

$\phi = 2\pi/3$

12. A current carrying loop of radius a is placed in $X - Y$ Plane with its center at origin. Find magnetic field on the point $(0,0,a)$?

A. $\frac{\mu_0 i}{2\sqrt{2}\,a}$
B. $\frac{\mu_0 i}{4\sqrt{2}\,a}$
C. $\frac{2\mu_0 i}{a}$
D. $\frac{\mu_0 i}{4a}$



Answer (B)

Solution:

For a current carrying loop magnetic field can be given as:

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{\mu}}{(a^2 + z^2)^{3/2}}$$

Magnetic moment:

$$|\vec{\mu}| = (\pi a^2) i$$

$$\vec{B} = \frac{\mu_0}{2} \cdot \frac{a^2 i}{(a^2 + z^2)^{3/2}}$$

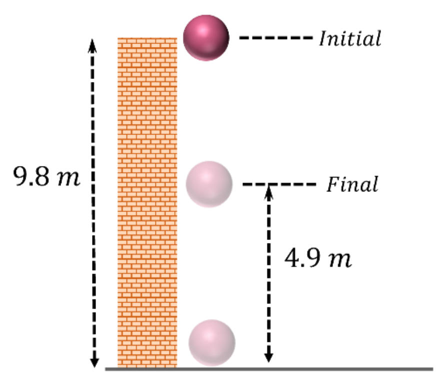
For $z = a$;

$$\vec{B} = \frac{\mu_0}{2} \cdot \frac{a^2 i}{(2a^2)^{3/2}} = \frac{\mu_0 i}{4\sqrt{2} a}.$$

13. A ball of mass $2\ kg$ is dropped from a height of $9.8\ m$ and rebounds to a height of $4.9\ m$. If it remains in contact with ground for $0.2\ seconds$, the average force on the ball by the ground is $x(\sqrt{2} + 1)\ Newtons$. Find x (Take $g = 9.8\ m/s^2$)

Answer (98)

Solution:



When ball hits the ground, velocity of ball is:

$$v_{initial} = \sqrt{2g(9.8)}$$

Velocity of ball after hitting the ground is:

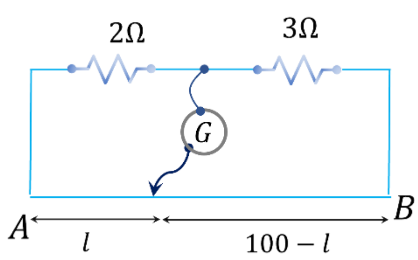
$$v_{final} = \sqrt{2g(4.9)}$$

$$F\Delta t = m(v_{final} - v_{initial})$$

$$F(0.2) = 2 \times (\sqrt{2 \times 9.8 \times 9.8} - (-\sqrt{2 \times 9.8 \times 4.9}))$$

$$F = 98\ (\sqrt{2} + 1)$$

14. Consider the meter bridge setup shown:



If a shunt resistance $x\ \Omega$ is added to 3Ω resistor, balance point shift by $22.5\ cm$. Find x

Answer (2)

Solution:

When a resistance of $x\ \Omega$ connected in parallel with $3\ \Omega$ resistance. Effective resistance becomes less than $3\ \Omega$. So, balance point shifts to right

For balanced bridge:

$$\frac{2}{3} = \frac{l}{100 - l}$$

$$l = 40\ cm$$

$$\frac{2}{R} = \frac{62.5}{37.5}, where\ R = \frac{3x}{3+x}$$

$$\frac{3x}{3+x} = \frac{75}{62.5} \Rightarrow x = 2\ \Omega$$

15. If dimensional formula of pressure gradient is X, Electric field has Y, Energy density has W and Latent heat has Z. Find dimensional formula of $\frac{[X][Y]}{[Z][W]}$

A. $[ML^{-2}T^{-1}A^{1}]$
B. $[ML^{-2}T^{-1}A^{-1}]$
C. $[M^{-1}L^{2}T^{-1}A^{1}]$
D. $[ML^{2}T^{-1}A^{-1}]$

Answer (B)

Solution:

$$[X] = \left[\frac{\Delta P}{\Delta X}\right] = \left[\frac{MLT^{-2}}{L^3}\right] = [ML^{-2}T^{-2}]$$

$$[Y] = [E] = [MLT^{-3}A^{-1}]$$

$$[W] = \left[\frac{MLT^{-2}}{L^2}\right] = [ML^{-1}T^{-2}]$$

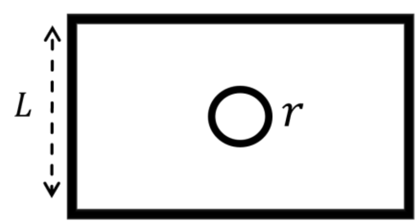
$$[Z] = \left[\frac{ML^2T^{-2}}{M}\right] = [L^2T^{-2}]$$

$$\frac{[X][Y]}{[Z][W]} = \left[\frac{[ML^{-2}T^{-2}][MLT^{-3}A^{-1}]}{[L^2T^{-2}][ML^{-1}T^{-2}]}\right]$$

$$= [ML^{-2}T^{-1}A^{-1}]$$

16. A small circular loop of radius r is placed in the plane of a square loop of side length $L (r \ll L)$. Circular loop is at the center of square as shown in the figure. Find mutual inductance.

A. $\frac{\mu_0 r^2}{\sqrt{2}L}$
B. $\frac{\pi \mu_0 r^2}{2L}$
C. $\frac{2\sqrt{2}\mu_0 r^2}{L}$
D. $\frac{4\mu_0 r^2}{L}$



Answer (C)

Solution:

B_{center} of rectangular loop

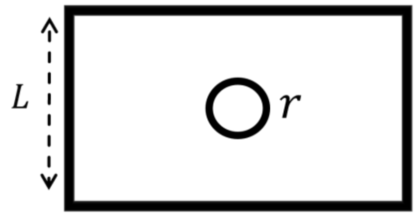
$$= \frac{\mu_0 i}{4\pi(\frac{L}{2})}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right] \times 4$$

$$= \frac{4\mu_0 i}{\sqrt{2}\pi L}$$

$$= \frac{2\sqrt{2}\mu_0 i}{\pi L}$$

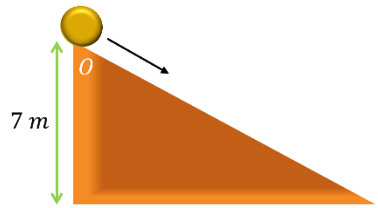
Flux in circular loop, $(\phi) = \pi r^2 \times B$

Self-inductance $= \frac{\phi}{i} = \frac{2\sqrt{2}\mu_0 r^2}{L}$



17. A solid sphere is released from point O at the top of an incline as shown. Find the value of velocity of centre of mass of sphere at the bottom most point of the incline after it reaches there doing pure rolling $(g = 10\ m/s^2)$

A. $3\ m/s$
B. $7\ m/s$
C. $10\ m/s$
D. $0.7\ m/s$



Answer (C)

Solution:

Using energy conservation

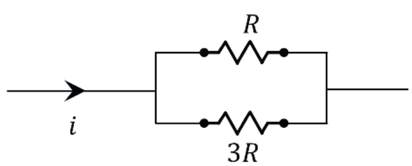
$$\frac{1}{2}mv_{cm}^2 + \frac{1}{2}\frac{2}{5}mR^2\left(\frac{v_{cm}}{R}\right)^2 = mgh$$

$$\frac{7}{10}mv_{cm}^2 = mgh$$

$$v_{cm} = \sqrt{\frac{10}{7}gh} = 10\ m/s$$

18. In the part of a circuit shown, find the ratio of rate of heat produced in R to that in $3R$.

A. $1:9$
B. $1:3$
C. $3:1$
D. $9:1$



Answer (C)

Solution:

$$Heat\ Produced(H) = \frac{V^2}{R}$$

Since V is same,

$$\frac{H_R}{H_{3R}} = \frac{3R}{R}$$

$$H_R : H_{3R} = 3:1$$

19. A disk of radius R is given by ω_o angular speed and placed gently on a rough horizontal surface. Find the velocity of center of disk when pure rolling starts.

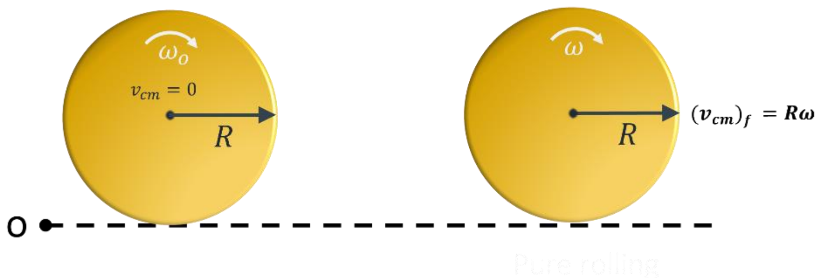
A. $R\omega_0/3$

B. $R\omega_o$

C. $R\omega_o/4$
D. $2R\omega_o$

Answer (A)

Solution:



Applying angular momentum conservation about point 'O'

$$I_{CM}\omega_0 = (I_{CM} + MR^2)\omega$$

$$\frac{1}{2}MR^2\omega_0 = \frac{3}{2}MR^2\omega$$

$$\omega_0 = 3\omega$$

$$\Rightarrow (v_{CM})_f = R\omega = \frac{R\omega_o}{3}$$

20. In a standard YDSE first minima is obtained in front of the slit for $\lambda = 800\ nm$. If the distance between the slit and screen is $5\ m$ then separation between the slits is equal to

A. $5 \times 10^{-2}\ m$
B. $5\ mm$
C. $3\ mm$
D. $2\ mm$

Answer (D)

Solution:

$$\frac{d}{2} = \frac{\lambda D}{2d}$$

$$d = \sqrt{\lambda D} = \sqrt{800 \times 10^{-9} \times 5} = 2\ mm$$

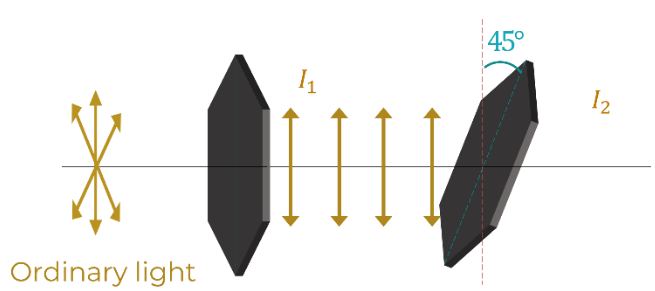
21. Two polarizers P_1 and P_2 are placed such that their transmission axis are at $45°$ from each other. Ordinary light is passed through P_1, I_1 intensity is observed and when this light is passed through P_2, I_2 intensity is observed. Find I_1/I_2 ?

Answer (2)

Solution:

$$I_2 = I_1 \cos^2\theta = I_1 \cos^2 45°$$

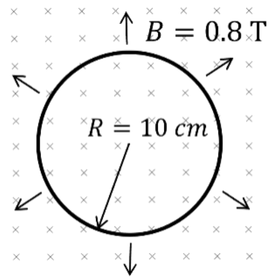
$$\frac{I_1}{I_2} = \frac{1}{\cos^2 45°} = 2$$



22. Magnetic field through a circular loop is $0.8\ T$. The radius of loop is expanding at $2\ cm/s$. The induced emf in the loop, when radius of the loop is $10\ cm$, is $x\pi \times 10^{-4}\ volts$. Find x.

Answer (32)

Solution:



$$\phi = B(\pi r^2)$$

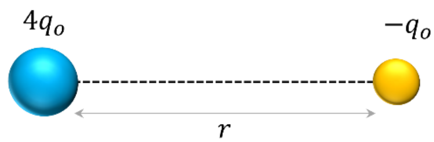
$$\varepsilon_{ind} = \left|\frac{d\phi}{dt}\right| = \frac{d(B\pi r^2)}{dt} = B\pi \times 2r\frac{dr}{dt}$$

$$= \pi \times 0.8 \times 2 \times \frac{10}{100} \times \frac{2}{100}$$

$$= 32 \times 10^{-4}\ Volts$$

23. Two point charges are arranged as shown:

Find the distance from $4q_o$ where net electric field is zero.



A. $4r$

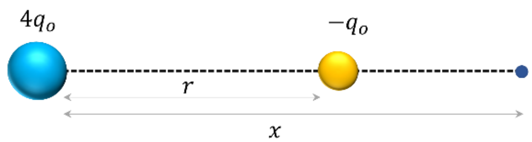
B. $3r$

C. $r/2$

D. $2r$

Answer (D)

Solution:



$$\frac{1}{4\pi\varepsilon_o}\frac{4q_o}{x^2} = \frac{1}{4\pi\varepsilon_o}\frac{q_o}{(x-r)^2}$$

$$x^2 = 4(x-r)^2$$

By solving,

$\Rightarrow x = 2r$ and $x = 2r/3$ (at this point net electric field is not zero)